Varied effects of shear correction on thermal vibration of Functionally Graded Material plates
C.C. Hong*

*Department of Mechanical Engineering, Hsiuping University of Science and Technology, Taichung, 412-80 Taiwan, ROC

Abstract

The varied shear correction coefficient and the first-order shear deformation theory (FSDT) effects on the functionally graded material (FGM) thick plates under thermal vibration are investigated by using the generalized differential quadrature (GDQ) method. The computed and varied values of shear correction coefficient are usually functions of total thickness of plates, FGM power law index and environment temperature. The shear correction coefficient equation of FGM plates is derived and obtained by using the total strain energy principle. Two parametric: environment temperature and FGM power law index effects on the stress and deflection of FGM thick plates are obtained and investigated. Further application of suitability might focus on the experiment of physicals.

DOI:https://doi.org/10.24243/JMEB/2.1.156

1 Introduction

There are some main explanations and fundamental theoretical backgrounds of using on shear correction, thermal vibration and functionally graded material (FGM) plates. The effect of shear correction on the tangential displacements of thick plates can be considered in linear and nonlinear forms. The effect of thermal vibration on the stresses of layers with temperature difference can be considered in an important item. The FGM plates usually composed with the metal and ceramic properties of constituent materials. There are some investigations on the FGM plates. In 2014, Najafabadi et al. [1] used the analytical modeling for the heat conduction equation to design the failure prevention under transient thermal loading in a FGM slab symmetrically surface heated, the hyperbolic model is found in more appropriate than the classical Fourier model. In 2014, Belabed et al. [2] used an efficient, simple higher order shear and normal deformation theory to predict the bending and free vibration responses of FGM plates. In 2014, Li et al. [3] used the method of fundamental solutions (MFS) to obtain the fundamental solutions of transient three-dimensional (3D) heat conduction in FGM. In 2014, Taheri et al. [4] provided an optimization structure design for FGM under thermal and mechanical loadings by using an isogeometrical approach. In 2014, Akavci [5] presented the free vibration analysis of FGM plates on elastic foundation including the transverse shear deformations effect. In 2014, Thai et al. [6] presented the free vibration analysis of FGM sandwich plates by using a first-order shear deformation theory (FSDT). In 2014, Xiang et al. [7] found the natural frequency of the FGM plates on elastic foundations by using an nth-order shear deformation theory and a mesh-less approach. In 2013, Jha et al. [8] presented free vibration analyses of FGM plates under severe thermo-mechanical loading by using a higher order shear and normal

*Corresponding Author : CC. Hong
Email Address: cchong@mail.hust.edu.tw
Tel: +886-919037599
deformations theory. In 2011, Fu et al. [9] used the boundary knot method (BKM) to solve the heat conduction in nonlinear FGM problems. In 2008, Nguyen et al. [10] presented the static analysis for FGM plate included the first-order shear deformation models. There are some GDQ computational experiences in the composited FGM shells and plates. In 2014, Hong [11] presented the thermal vibration and transient response of Terfenol-D FGM plates by considering the FSDT model and the varied modified shear correction factor effects. It is interesting to investigate the thermal stresses and center deflection of GDQ computation in this FSDT and the varied effects of shear correction coefficient of FGM thick plates with four edges in simply supported boundary conditions. Environment temperature and FGM power law index effects on the thermal stress and center deflection of FGM thick plates are also obtained and investigated.

2 Formulations

For a two-material FGM plate, the material properties of power-law function of FGM plates are considered with Young’s modulus $E_{fgm}$ of FGM in standard variation form of power law index $R_n$, the others are assumed in the simple average form [12]. The properties $P_i$ of individual constituent material of FGMs are functions of environment temperature $T$.

The time dependence of displacements $u$, $v$ and $w$ of thick plates are assumed in the following linear FSDT equations [13].

$$
\begin{align*}
u &= u^0(x, y, t) + z \psi_x(x, y, t) \\
v &= v^0(x, y, t) + z \psi_y(x, y, t) \\
w &= w(x, y, t)
\end{align*}
$$

(1) (2) (3)

where $u^0$ and $v^0$ are displacements in the $x$ and $y$ axes direction, respectively, $w$ is transverse displacement in the $z$ axis direction of the middle-plane of plates, $\psi_x$ and $\psi_y$ are the shear rotations, $t$ is time.

For the normal stresses ($\sigma_x$ and $\sigma_y$) and the shear stresses ($\sigma_{xy}$, $\sigma_{yz}$ and $\sigma_{xz}$) in the thick FGM plate under temperature difference $\Delta T$ for the $k$ th layer are in the following equations [14], [15].

$$
\begin{align*}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_{xy}
\end{bmatrix}_{(k)} &= \begin{bmatrix}
\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\
\overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\
\overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66}_{(k)}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x - \alpha_x \Delta T \\
\varepsilon_y - \alpha_y \Delta T \\
\varepsilon_{xy} - \alpha_{xy} \Delta T_{(k)}
\end{bmatrix}
\end{align*}
$$

(4)

$$
\begin{align*}
\begin{bmatrix}
\sigma_{yz} \\
\sigma_{xz}
\end{bmatrix}_{(k)} &= \begin{bmatrix}
\overline{Q}_{44} & \overline{Q}_{45} \\
\overline{Q}_{45} & \overline{Q}_{55}_{(k)}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{yz} \\
\varepsilon_{xz}
\end{bmatrix}_{(k)}
\end{align*}
$$

(5)

where $\alpha_x$ and $\alpha_y$ are the coefficients of thermal expansion, $\alpha_{xy}$ is the coefficient of thermal shear, $\overline{Q}_{ij}$ is the stiffness of FGM plates. $\varepsilon_x$, $\varepsilon_y$ and $\varepsilon_{xy}$ are in-plane strains, not negligible $\varepsilon_{yz}$ and $\varepsilon_{xz}$ are shear strains. The temperature difference between the FGM plate and curing area is given in the following equation.

$$
\Delta T = T_0(x, y, t) + \frac{z}{h^*}T_1(x, y, t)
$$

(6)

in which $T_0$ and $T_1$ are temperature parameters in functions of $x$, $y$ and $t$. $h^*$ is the total thickness of FGM plates.
The dynamic equations of motion for a plate are introduced by Whitney in 1987 [15]. The dynamic equilibrium differential equations of FGM plates in terms of matrix coefficients \((A_{ij}, B_{ij}, D_{ij}) = \frac{1}{2} \int_{-h}^{h} \bar{Q}_{ij}(l, z, z^2) dz, (i, j = 1, 2, 6)\),

\[
A_{i,j} = \int_{-h}^{h} k_{i,j} \bar{Q}_{i,j} dz, (i, j = 4, 5) \quad \text{in partial derivatives of displacements and shear rotations subjected to partial derivatives of thermal loads} \left(\bar{N}, \bar{M}\right), \text{mechanical loads} \left(p_1, p_2, q\right) \text{and inertia terms} \left(\rho, H, I\right) \text{can be obtained. In which} k_{i,j} \text{is the shear correction coefficient. The values of} k_{i,j} \text{are usually functions of} h^*, T \text{and} R_n. \text{The} \bar{Q}_{ij} \text{and} \bar{Q}_{i,j} \text{for FGM plates are used in the simple forms in 2007 by Shen} [16] \text{to calculate the stresses and} A_{ij}.

Usually the value of shear correction coefficient is not constant in the FGM plates. The modified shear correction factor \(k_{i,j}\) can be derived and based on the energy equivalence principle [15] by equaling the total strain energy equation due to transverse shears and assumed due to shear forces with \(k_{i,j}\), thus the \(k_{i,j}\) equation can be obtained for the FGM plates in the following [11].

\[
k_{i,j} = \frac{1}{h^*} FGMZSV, \tag{7}
\]

in which \(FGMZSV\) and \(FGMZIV\) are the parameters in functions of \(\nu_{fgm}, h^*, R_n, E_1\) and \(E_2\) with \(E_1\) and \(E_2\) are the Young’s modulus of the constituent material 1 and 2 of FGM, respectively.

The GDQ method [11], [17], [18] approximates the derivative of function is applied in the formulation of dynamic equilibrium differential equations by considering for four sides simply supported, not symmetric, orthotropic of laminated FGM plates, the dynamic GDQ discrete equations in matrix notation can be derived.

3. Numerical Results and Discussions

The following coordinates \(x_i\) and \(y_j\) for the grid points numbers \(N\) and \(M\) of FGM thick plates are used to study the GDQ results of varied shear correction coefficient calculations with plates layers in the stacking sequence \(0^\circ/0^\circ\), under four sides simply supported boundary condition, no in-plane distributed forces \((p_1 = p_2 = 0)\) and no external pressure load \((q = 0)\).

\[
x_i = 0.5 \{1 - \cos(\frac{i-1}{N-1} \pi)\} a, i = 1, 2, ..., N, \tag{8}
\]

\[
y_j = 0.5 \{1 - \cos(\frac{j-1}{M-1} \pi)\} b, j = 1, 2, ..., M. \tag{9}
\]

The time sinusoidal displacement and temperature of thermal vibrations are used as follows.

\[
u = [u^0(x, y) + z \psi_x(x, y)] \sin(\omega_{mn} t), \tag{10}
\]

\[
v = [v^0(x, y) + z \psi_y(x, y)] \sin(\omega_{mn} t), \tag{11}
\]

\[
w = w(x, y) \sin(\omega_{mn} t). \tag{12}
\]

\[
\Delta T = [T_0(x, y) + \frac{z}{h^*} T_1(x, y)] \sin(\gamma t). \tag{13}
\]

And with the simple vibration of temperature parameter
\[ T_0(x, y) = 0, \quad \text{(14)} \]
\[ T_1(x, y) = \overline{T_1} \sin(\pi x / a) \sin(\pi y / b). \quad \text{(15)} \]

in which \( \omega_{nm} \) is the natural frequency in mode shape numbers \( m \) and \( n \) of the plates, \( \gamma \) is the frequency of applied heat flux, \( \overline{T_1} \) is the amplitude of temperature.

The constituent material 1 of FGM is SUS304 (stainless steel), the constituent material 2 of FGM is \( Si_3N_4 \) (silicon nitride) used for the numerical GDQ computations, with constituent material 1 thickness \( h_1 \) and constituent material 2 thickness \( h_2 \). Firstly, the dynamic convergence study of center deflection amplitude \( w(a/2, b/2) \) (unit mm) in FGM plates are obtained in Table 1 by considering the varied effects of shear correction coefficient and \( h^* = 1.2 \text{ mm}, \ h_1 = h_2 = 0.6 \text{ mm under } R_n = 1, \ m = n = 1, \) calculated \( k_\alpha = 0.149001 \) with \( T = 100^\circ K \) and \( \overline{T_1} = 100^\circ K \) at \( t = 6s \). The error accuracy is 1.66e-04 for the center deflection amplitude of \( a/b = 2 \) and \( a/h^* = 5 \). The 17x17 grid point can be treated in the convergence result and used in the following GDQ computations of time responses for deflection and stress of FGM plates. There are quite great different \( w(a/2, b/2) \) values between the calculated and the commonly used of \( k_\alpha = 5/6 = 0.833333 \). So it is interesting to consider the effects of calculated \( k_\alpha \) on the FGM plates. In the FGM plates \( (B_y \neq 0) \), varied values of \( k_\alpha \) are usually functions of \( h^*, R_n \) and \( T \). For \( a/h^* = 5, \ a/b = 1, \ h^* \) from 0.12 mm to 4.8 mm, \( h_1 = h_2 \), calculated values of \( k_\alpha \) under \( T = 653^\circ K \) are shown in Table 2 used for the GDQ and shear calculations. For \( h^* = 1.2 \text{ mm} \), values of \( k_\alpha \) (from 0.678557E-01 to 0.447611 under \( T = 653^\circ K \) ) are increasing with \( R_n \) (from 0.1 to 10).

**Table 1 Dynamic convergence of FGM plates with calculated \( k_\alpha = 0.149001 \)**

<table>
<thead>
<tr>
<th>( a/h^* )</th>
<th>( N \times M )</th>
<th>( a/b = 0.5 )</th>
<th>( a/b = 1 )</th>
<th>( a/b = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>13x13</td>
<td>-0.25381E-07</td>
<td>-0.36329E-07</td>
<td>-0.80167E-07</td>
</tr>
<tr>
<td>15x15</td>
<td>-0.25381E-07</td>
<td>-0.36329E-07</td>
<td>-0.80167E-07</td>
<td></td>
</tr>
<tr>
<td>17x17</td>
<td>-0.25381E-07</td>
<td>-0.36327E-07</td>
<td>-0.80163E-07</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>13x13</td>
<td>-0.13960E-05</td>
<td>-0.22062E-05</td>
<td>-0.47090E-05</td>
</tr>
<tr>
<td>15x15</td>
<td>-0.13960E-05</td>
<td>-0.20327E-05</td>
<td>-0.47431E-05</td>
<td></td>
</tr>
<tr>
<td>17x17</td>
<td>-0.13955E-05</td>
<td>-0.20399E-05</td>
<td>-0.47516E-05</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>13x13</td>
<td>-0.29399E-05</td>
<td>-0.43410E-05</td>
<td>-0.10398E-04</td>
</tr>
<tr>
<td>15x15</td>
<td>-0.29399E-05</td>
<td>-0.42779E-05</td>
<td>-0.10446E-04</td>
<td></td>
</tr>
<tr>
<td>17x17</td>
<td>-0.29471E-05</td>
<td>-0.43368E-05</td>
<td>-0.10457E-04</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>13x13</td>
<td>-0.49793E-05</td>
<td>-0.75291E-05</td>
<td>-0.30096E-04</td>
</tr>
<tr>
<td>15x15</td>
<td>-0.49878E-05</td>
<td>-0.74580E-05</td>
<td>-0.29849E-04</td>
<td></td>
</tr>
<tr>
<td>17x17</td>
<td>-0.49793E-05</td>
<td>-0.75263E-05</td>
<td>-0.29889E-04</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>13x13</td>
<td>-0.17456E-04</td>
<td>-0.33956E-04</td>
<td>0.11457E-02</td>
</tr>
<tr>
<td>15x15</td>
<td>-0.17208E-04</td>
<td>-0.32719E-04</td>
<td>0.11447E-02</td>
<td></td>
</tr>
<tr>
<td>17x17</td>
<td>-0.17541E-04</td>
<td>-0.32602E-04</td>
<td>0.11449E-02</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2 Varied shear correction coefficient \( k_\alpha \) vs. \( R_n \) under \( T = 653^\circ K \)**

<table>
<thead>
<tr>
<th>( h^* ) (mm)</th>
<th>( k_\alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>0.678557E-01</td>
</tr>
<tr>
<td>0.36</td>
<td>0.80167E-01</td>
</tr>
<tr>
<td>0.60</td>
<td>0.92480E-01</td>
</tr>
<tr>
<td>0.84</td>
<td>1.0470E-01</td>
</tr>
<tr>
<td>1.2</td>
<td>0.10446E-04</td>
</tr>
<tr>
<td>1.44</td>
<td>0.10457E-04</td>
</tr>
<tr>
<td>1.68</td>
<td>0.10457E-04</td>
</tr>
<tr>
<td>2.1</td>
<td>0.10398E-04</td>
</tr>
<tr>
<td>2.52</td>
<td>0.10447E-02</td>
</tr>
<tr>
<td>2.94</td>
<td>0.11457E-02</td>
</tr>
<tr>
<td>3.36</td>
<td>0.11447E-02</td>
</tr>
</tbody>
</table>
Figure 2 shows the center deflection amplitude $w(a/2, b/2)$ (unit mm) versus $T$ for all different values $R_n$ (from 0.1 to 10) of FGM plates calculated and varied values of $k_a$, for $L/h^* = 5$, $a/b = 1$, $h^* = 1.2$ mm, $h_1 = h_2 = 0.6$ mm, $T = 100^\circ K$, at $t = 3s$. The maximum value of center deflection amplitude is 0.0198045 mm occurs at $T = 653^\circ K$ for $R_n = 5$. The center deflection amplitude values are all decreasing versus $T$ from $T = 653^\circ K$ to $T = 1000^\circ K$, for $R_n = 2$, 5 and 10, they can withstand for higher temperature ($T = 1000^\circ K$) of environment. The center deflection amplitude values are almost keep constant versus $T$ for the others $R_n = 0.1, 0.2, 0.5$ and 1, because the main effect of the Young’s modulus on these $R_n$ are in small value of changes.

Figure 3 shows the dominated stresses $\sigma_x$ (unit $GPa$) at center position of outer surface $z = 0.5h^*$ versus $T$ for all different values $R_n$ of FGM plates as the analyses of deflection case in Fig. 2. The absolute values of dominated stresses $\sigma_x$ versus $T$ are all increasing (from $T = 100^\circ K$ to $T = 653^\circ K$) and then decreasing (from $T = 653^\circ K$ to $T = 1000^\circ K$) for $R_n = 5$, but all decreasing (from $T = 100^\circ K$ to $T = 653^\circ K$) and then increasing (from $T = 653^\circ K$ to $T = 1000^\circ K$) for $R_n = 10$. Each keeping almost constant stresses values versus $T$ for others values of $R_n$. 

<table>
<thead>
<tr>
<th>$R_n = 0.1$</th>
<th>$R_n = 0.2$</th>
<th>$R_n = 0.5$</th>
<th>$R_n = 1$</th>
<th>$R_n = 2$</th>
<th>$R_n = 5$</th>
<th>$R_n = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>0.108966</td>
<td>0.231570</td>
<td>0.591461</td>
<td>1.00646</td>
<td>0.931476</td>
<td>0.837602</td>
</tr>
<tr>
<td>1.2</td>
<td>0.678557E-01</td>
<td>0.718781E-01</td>
<td>0.866672E-01</td>
<td>0.117077</td>
<td>0.185741</td>
<td>0.351063</td>
</tr>
<tr>
<td>2.4</td>
<td>0.636415E-01</td>
<td>0.632094E-01</td>
<td>0.628452E-01</td>
<td>0.619070E-01</td>
<td>0.535113E-01</td>
<td>0.172796E-01</td>
</tr>
<tr>
<td>4.8</td>
<td>0.596697E-01</td>
<td>0.555181E-01</td>
<td>0.452845E-01</td>
<td>0.318705E-01</td>
<td>0.400004E-01</td>
<td>0.619180E-03</td>
</tr>
</tbody>
</table>
4. Conclusions

The GDQ solutions are calculated and investigated for the deflections and stresses in the thermal vibration of FGM thick plates by considering the varied effects of shear correction coefficient. GDQ results show: (a) varied values of $\alpha_k$ are usually functions of $h^*$, $R_n$ and $T$. (b) The maximum value of center deflection amplitude is 0.163142mm occurs at $t = 0.001s$ for thick $a/h^* = 5$ at $T = 653^\circ K$. (c) The center deflection amplitude values are all decreasing versus $T$ from $T = 653^\circ K$ to $T = 1000^\circ K$, for $R_n = 2, 5$ and 10, they can withstand for higher temperature ($T = 1000^\circ K$) of environment.

References


Jha DK, Kant T, Srinivas K, Singh RK. An accurate higher order displacement model with shear and normal deformations effects for functionally graded plates. Fusion Engineering and Design. 2013 Dec 31;88(12):3199-204.


