Identification of transverse elastic properties of the diaphysis of cortical bone

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Abstract

An inverse identification method is proposed to determine transverse elastic properties of cortical bone of diaphysis of long bones. Compression tests on tubular diaphysis disks are carried out. The approach couples the digital image correlation technique with the finite element updating method. An optimisation algorithm is used to minimize the numerical-experimental response for the elastic properties of the model. Diaphysis of tibia, femur and humerus is tested. In a first approximation, cortical bone tissue is considered transversely isotropic. A transverse elastic modulus of 8.2, 8.9 and 7.0 GPa is identified for tibia, femur and humerus tissue, respectively, with a typical scatter for biological materials. The response of the diaphysis disk to compression loading is however less sensitive to the Poisson’s ratio; hence, this parameter is more difficult to evaluate in practice for the proposed configuration. DOI: https://doi.org/10.24243/JMEB/2.5.172

1. Introduction

Cortical bone tissue is a composite material with a complex, heterogeneous and hierarchical structure. Moreover, its composition and structure vary spatially and temporally, depending on both mechanical and physiological environment. Therefore, the mechanobiology of this material, give rise to difficulties in measuring constitutive parameters that govern the mechanical behaviour, including elastic properties. The correct identification of these properties through appropriate methodologies is nevertheless essential. Classical mechanical tests, generating homogenous stress and strain fields in the region of interest, have been proposed for the characterisation of the mechanical properties of this type of tissue [1]. However, thanks to the advent of optical techniques, digital cameras, and processing algorithms, it is now possible to highlight a new approach to mechanical testing. In this identification strategy the material is submitted to complex and heterogeneous stress/strain fields over the region of interest, in a way that several material parameters can be activated in the mechanical response. In this framework, however, there is no explicit relationship linking geometry, applied forces, and deformations to the unknown mechanical properties. It is therefore necessary to use suitable inverse methods, such as the finite element updating method (FEMU) [2] or the virtual field method (VFM) [3], for determining the whole set of constitutive parameters from single mechanical tests.

In this work, a numerical-experimental identification method was proposed based on a compression test over transversal sections of diaphysis of long bones (tibia, femur and humerus). The approach combines full-field deformation measurements obtained by digital image correlation (DIC) with FEMU. Specimen-specific geometrical...
and finite element meshes were generated. Relevant elastic constitutive parameters of cortical bone tissue in the transversal plane were determined, assuming, in a first approximation, a transversely isotropic mechanical behaviour of the diaphysis. This means that only two elastic parameters on the radial-tangential (RT) plane are independent. Firstly, the validation of the identification method was assessed numerically. In this case, elastic reference properties were used for the bone tissue. Secondly, the identification algorithm, based on a given objective function written in terms of displacement fields was carried out on experimental data for the compression tests.

2. Methods and Materials

2.1 Specimens and diametral compression tests

Cortical bone tissue of goat was used in this work. Specimens were cut from diaphysis of long bone tissues of tibia, femur and humerus, with a nominal thickness of 10 mm. Irregular tubular disks were obtained as shown in Fig. 1. The tubular specimens were tested in compression. The tests were performed on an INSTRON 5848 MicroTester universal test machine (Model 5848, 101 Instron, Barcelona, Spain), with displacement control at 0.3 mm/min (Fig. 1).

2.2 Digital image correlation

DIC provides full-field displacements of a target object by correlating images recorded before and after a given deformation [4]. It is assumed herein that images are grabbed by a monovision camera-lens optical system (DIC-2D). To solve the correspondence problem in image processing, DIC requires that the surface of interest has a random, textured pattern uniquely characterising the material surface. The reference (or undeformed) image is typically divided into subsets, with suitable isotropy and contrast. This will define the spatial resolution (Δu) and resolution (σu) associated to DIC measurements. Therefore, they must be carefully chosen with regard to the application, in a compromise between correlation (small subsets) and interpolation (large subsets) errors. Several mathematical correlation criteria have been proposed for estimation of the displacement fields in the subset matching process [5]. It has been shown that the zero-normalized sum of squared differences (ZNSSD) is a robust algorithm since it take into account offset and linear scale variations of light intensity and is the most efficient when using iterative procedure for the optimisation problem [5].

DIC provides displacements at a large set of discrete data points across a region of interest. However, continuous strain fields are usually required in material parameter characterisation. Therefore, a suitable technique is needed to calculate the strain field from the measured displacement field. It is worth noticing that the numerical differentiation of the measured displacement fields is not straightforward since this procedure can amplify noise, inherently present in the measurements. For instance, direct differentiation using finite differences can lead to a strain resolution in the range of 10⁻³ (e.g., for a displacement resolution of about 10⁻² and a strain step of 5 subsets, a strain resolution of 2×10⁻³ is obtained using central finite differences), which is normally too high for practical use in mechanical tests. Several strategies can be then used most of them consists in approximating the data points using smooth basis functions. The differentiation of the data is then based on the differentiation of the approximated basis functions in the least-square sense. Generically these methods can be sorted on global and local strategies. In this work, point-wise local least-squares fitting was used [6]. This approach used a first order shape function to locally approximate the displacement field measured by DIC. In this approach the regularisation parameters is the size of the strain window: (2m+1)×(2m+1). The value of m must be chosen in a compromise between level of low-pass filtering and accuracy of the representativeness of the strain field.

The natural surface of the bone specimen was prepared for DIC measurements. For that purpose, a speckled pattern was created by spreading matt black paint over the material surface using an airbrush (Fig. 1). Throughout the tests, synchronised images of the specimen deformation were recorded using a Pike F-120C camera coupled to a Nikon AF Nikkor lens 28-105 mm. The Ncorr v1.2 open-source 2D digital image correlation code was used for image correlation [7]. The spatial displacement resolution associated with the measurements was 0.3 mm, corresponding to a correlation window of 15 pixels. The magnification of the optical system yields a conversion factor of 20 μm/pixel. For
the reconstruction of the strain field, a local derivation approach was used, on a mesh of 5 subset-data points, corresponding to a spatial resolution of 0.1 mm. The resolution in displacement and deformation associated with the measurements is of the order of 0.01 pixel and 0.02%, respectively [8].

2.3 Finite element model

An image-based geometry of the FE model for each specimen was defined from the reference image recorded using the DIC optical system. A FE mesh was then generated for each specimen specific geometry. The ANSYS Academic code was finally used to build the FE model of the compression test [9]. The four-node isoperimetric quadrilateral plane element was selected from the elements library. The plane stress state through the specimen thickness was assumed. The convergence of the mesh was achieved for an element size of 0.5 mm (Fig. 1). An isotropic linear elastic model with reference properties of $E = 9 \text{ GPa}$ and $\nu = 0.26$ was assumed. The boundary conditions were simulated considering the specimen-platen contact, using CONTA172 and TARGET169 elements (Fig. 1). A coefficient of friction equal to 0.5 was considered. The prescribed displacements were defined considering the displacement experimentally applied within the linear elastic domain.

2.4 Optimisation algorithm

In the proposed FEMU approach, the identification of the elastic properties is achieved by solving an optimisation problem. This consists typically in minimizing the gap between the experimental mechanical response and that estimated by the numerical model of the mechanical test. The objective function can be written in terms of displacements, deformations and stress (assuming a given constitutive model) or a combination between these quantities [10]:

\[
\begin{align*}
\text{FO1} &= \frac{1}{N} \sum_{i=1}^{N} \left[ \frac{u_i^{\text{exp}} - u_i^{\text{num}}}{u_{\text{max}}} + \frac{v_i^{\text{exp}} - v_i^{\text{num}}}{v_{\text{max}}} \right] \\
\text{FO2} &= \sqrt{\left( \varepsilon_{xx,i}^{\text{exp}} - \varepsilon_{xx,i}^{\text{num}} \right)^2 + \left( \varepsilon_{yy,i}^{\text{exp}} - \varepsilon_{yy,i}^{\text{num}} \right)^2} \\
&\quad + \left( \varepsilon_{xy,i}^{\text{exp}} - \varepsilon_{xy,i}^{\text{num}} \right)^2 \\
\text{FO3} &= \sqrt{\left( \sigma_{xx,i}^{\text{exp}} - \sigma_{xx,i}^{\text{num}} \right)^2 + \left( \sigma_{yy,i}^{\text{exp}} - \sigma_{yy,i}^{\text{num}} \right)^2} \\
&\quad + \left( \sigma_{xy,i}^{\text{exp}} - \sigma_{xy,i}^{\text{num}} \right)^2
\end{align*}
\]
Fig. 2(left) shows the flowchart of the implemented FEMU algorithm. The research domains for each design parameter, $E$ and $\nu$, were within the intervals of $[3000; 11000]$ MPa and $[0.1; 0.4]$, respectively. To start the algorithm, arbitrary values of 6000 MPa and 0.2 were selected. The optimisation algorithm was based on the Nelder-Mead method that allows the minimization of strictly convergent functions [11].

3 Results

3.1 Numerical Validation

A numerical study was firstly proposed to validate the identification approach. In this case, the experimental observation was replaced by a reference numerical one, for a given set of elastic properties. The evaluation of the objective functions (OFs), in the optimisation algorithm described in Eq. (1), are shown in Fig 2 (right). As can be verified all the OFs can be represented by a convex function with local minimum. Fig. 3 illustrates the convergence of the elastic properties as a function of the iterations obtained by the optimisation algorithm. As can be seen, convergence occurs for both design variables ($E$, $\nu$), when compared to the reference values of the numerical model.

3.2 Experimental Identification

The proposed FEMU algorithm was then applied to experimental data. To start with, the force-displacement curves obtained for the compression tests of all specimens is shown in Fig 4 (left). In general, the curves are linear until failure. Fig 1(right) shows an example of the full-field strain components obtained experimentally and numerically (for an applied force value of 100 N). It is concluded that the strain components are not uniform throughout the surface of the tubular disk. This non-uniformity may have two sources: (1) the geometry of the specimen, which is not symmetrical with regard to the applied load; (2) the heterogeneity of the material.

4 Discussion

The application of the FEMU optimisation algorithm to the tubular tested specimen yield to the identification of the Young’s modulus as shown in Fig 4 (right). As can be seen, there is a convergence to typical values for this material and material plane (RT plane). Similar convergence curves were obtained for the Poisson’s ratio of the tested specimens, but systematically stabilizing after few iterations to a value of 0.1. This value is, however, lower than reference parameters [12]. Table 1 shows the results obtained by the FEMU identification method. As can be concluded, typical values are obtained for the Young’s modulus with a natural scatter for biological materials.

5 Conclusions

A disk compression test was proposed to retrieve elastic properties in the RT plane of cortical bone tissue. The DIC technique provided full-field deformation measurements throughout the whole region of the specimen. In this test there is no explicit relation between geometry, applied forces, kinematic quantities and elastic properties. Therefore, an inverse identification method was proposed, based on the finite element (FE) updating method. An optimisation algorithm for the isotropic elastic properties was implemented by minimizing the difference between the mechanical responses of a FE model and experimental observations. The modulus of elasticity was determined correctly with coefficients of variation typical for biological materials. However, the test configuration was in practice less sensitive to the Poisson’s ratio.

<table>
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<tr>
<th>Bone</th>
<th>$E$(MPa)</th>
<th>$\nu$(-)</th>
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<tr>
<td>Tibia</td>
<td>8213 ± 1363 (16.6%)</td>
<td>0.122± 0.03 (24.9%)</td>
</tr>
<tr>
<td>Femur</td>
<td>8879± 544 (6.1%)</td>
<td>0.1</td>
</tr>
<tr>
<td>Humerus</td>
<td>7888 ± 1311 (16.6%)</td>
<td>0.1</td>
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Fig. 2 (left) flowchart of the FEMU identification algorithm; (right) evaluation of the objective functions: numerical analysis

Fig. 3 Numerical convergence of elastic parameters for the three objective functions (bone reference properties: $E = 9$ GPa and $\nu = 0.26$).
Fig. 4 (left) load-displacement curves resulting from the compression tests; (right) convergence of the transversal Young’s modulus (Tibia, Femur and Humerus).

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